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Section 6.5
Laws of Logarithms

Definition → $\log_b x = y \iff x = b^y$

2. $\log_a 1 = 0, \quad \log_a a = 1$

3. $a^{\log_a m} = m$

4. $\log_a mn = \log_a m + \log_a N$

5. $\log_a \frac{m}{n} = \log_a m - \log_a N$

6. $\log_a x^r = r \cdot \log_a x$

7. $e^{x \ln a} = a^x$

$e^{\log_e a^x}$

8. $\log_a m$

(change of base)

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$\ln x = \log_e x$

$\log x = \log_{10} x$

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(14) $\log_2 2^{-13} = -13 \cdot \log_2 2 = -13(1) = -13$

(16) $\ln e^{\sqrt{2}} = \log_e e^{\sqrt{2}} = \sqrt{2} \cdot \log_e e = \sqrt{2}(1) = \sqrt{2} \quad (\ln e = 1)$

(18) $e^{\ln 8} = e^{\log_e 8} = 8$

(20) $\log_6 9 + \log_6 4 = \log_6 (9)(4) = \log_6 (36) = 2$

(22) $\log_8 16 - \log_8 2 = \log_8 \left(\frac{16}{2}\right) = \log_8 8 = 1$

(24) $\log_3 8 \cdot \log_8 9 = \frac{\log 8}{\log 3} \cdot \frac{\log 9}{\log 8} = \frac{\log 3^2}{\log 3} = \frac{2 \cdot \log 3}{\log 3} = 2$

(26) $5^{\log_5 6 + \log_5 7} = 5^{\log_5 42} = 42$

(28) $e^{\log_{e^2} 9} = \sqrt{9} = 3$ (ex.) $\rightarrow e^{\log_{\sqrt{e}} 7} = 7^2$

(ex.) $\rightarrow 2^{\log_4 7} = 2^{\log_{2^2} 7} = \sqrt{7}$

Expanding & Contracting

* Write as a sum or difference of logs.

$$\log_2 \frac{x^2 y^3}{z k^5} = \log_2 x^2 + \log_2 y^3 - \log_2 z - \log_2 k^5$$

$\sqrt[n]{x^m} = x^{m/n}$

$\sqrt[2]{x^1} = x^{1/2}$

$$= 2\log_2 x + 3\log_2 y - \log_2 z - 5\log_2 k$$

$$\log_6 \frac{\sqrt{x+1}\cdot(x-3)^5}{\sqrt[3]{x-2}\cdot(x+5)^{4/3}} = \log_6 \sqrt{x+1} + \log_6 (x-3)^5 - \log_6 \sqrt[3]{(x-2)} - \log_6 (x+5)^{4/3}$$

$$= \frac{1}{2}\log_6 (x+1) + 5\cdot \log_6 (x-3) - \frac{1}{3}\log_6 (x-2) - \frac{4}{3}\log_6 (x+5)$$

* Write as a single log

$$2\log_5 (x-3) - 5\log_5 (x+2) - \frac{1}{2}\log_5 (x+3) + \frac{2}{3}\log_5 (x-8)$$

$$= \log_5 (x-3)^2 - \log_5 (x+2)^5 - \log_5 (x+3)^{1/2} + \log_5 (x-8)^{2/3}$$

$$= \log_5 \frac{(x-3)^2 (x-8)^{2/3}}{(x+2)^5 (x+3)^{1/2}}$$